FORSYTH

On the construction and graduation of a mortality table from the United States census

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ON THE CONSTRUCTION AND GRADUATION OF A MORTALITY TABLE FROM THE UNITED STATES CENSUS

BY

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Chester H. Forsyth

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ON THE CONSTRUCTION AND GRADUATION OF A MORTALITY TABLE FROM THE UNITED STATES CENSUS.

by

Chester H. Forsyth

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Introduction

Before presenting the construction and graduation of the Mortality Table derived in the present paper, let me call especial attention to two main facts:

(a) as far as I can find, this is the first mortality table constructed from the statistics of the United States Census;

(b) the figures of this table are intended to represent, as nearly as possible, present day conditions in mortality rates.

Bearing on these two facts, the following brief discussion is given of (a) and (b):

(a) Several mortality tables have been constructed from incomplete data, such as death returns, physicians' reports, etc., but the Swedish Life Table by Dr Price in 1783 seems to be

the first national life table ever constructed from figures representing both the living and the dying. About nine tables of national and several of local character have been formed in England; six in Australia; three in Scotland; several in India; several in New Zealand; two in Ceylon; one in Canada, but none in the United States. Of course, several have been constructed from life insurance records, but the above information refers merely to what has been done with data from enumerations taken from people themselves.

(b) As to the change in mortality rates, I wish to quote, in part, the remarks of Oscar B Ireland before the fall meeting of the Actuarial Society of America in 1902 when he was president of that organization: " In my remarks at our meeting in May last, I had something to say about the mortality statistics to be gathered from the United States census reports, and noted that the twelfth census, taken in 1900, showed a falling off in the death rate of the registration area from 19.6 per 1000 in 1890 to 17.8 in 1900, and a diminution in the proportion of deaths from certain diseases that particularly affect children, with an increase in the proportion from several causes that are most operative late in life. And it may be noted that England, Scotland, Ireland, France, Germany, Austria, and Italy all showed a decline in the death rate for the decade beginning with 1890. Additional information as to this change in mortality rates in the United States has, since our last meeting, been given to the public, and is worth our attention.

In the first place, we have in Vol. III of the census report a table of death rates by ages under five, and by five-year

groups for ages from five upward, for the registration area; this this areauded 31.2 per cent of the whole population in 1890, and 37.9 per cent of the whole in 1900; the death rates given in this connection are believed to be " fairly reliable and comparable ". They show a decrease for evrey group up to age 59; under five years of age this decrease was 14.7 per thousand of population, and for the other eleven groups the average decrease was a little more than one per thousand. For ages 60 and above, every group shows an increase in the death rate; for twenty years the average increase is nearly 6 per thousand; while for the groups from age 80 upward the increases are 21.2, 25.8,,79.2, and 71.8 respectively. This very striking exhibit shows beyond dispute that a smaller proportion of deaths at very young ages, and a larger proportion of deaths at old ages occurred in the census year ending May 31 1900, than occurred ten years before. ----- Another showing of the relative increase in the number of persons at the older ages is in the form of a statement of the average age of the population of the United States, which is given at 24.6 years in 1880, 25.6 in 1890. 26.3 in 1900. And the median age, or age which has just half of the population on either side of it, has increased with surprising regularity for many years; for 1820 it is given as 16.7 years, and for 1900 as 22.8. Very recent statistics of the deaths in the state of Massachusetts harmonize with the general conditions above indicated, the average death for that state from 1892 to 1896 inclusive, was 19.7, and for 1897 to 1901 it was 17.6, while for the single year 1901 it was only 16.8. A large immigration, and advanced medical science, shown especially in the treatment of consumption, diptheria, and certain other diseases, are believed to be chief causes of these results."

Another writer says in a perhaps more popular vein: " In the last twenty-five years medical science has waged an unceasing and largely successful warfare against contagious diseases.----

The mortality from typhoid fever, scarlatina, and pneumonia has decreased nearly one half. Tuberculosis kills only two thirds as many as it did twenty years ago. Yellow fever has all but disappeared from the Southern ports in which it was formerly an annual scourge. Even so hopeless a malady as cerebro-spinal meningitis, which five years ago destroyed three out of every four of the children it assailed, now takes only one.--- Thus in Europe in the sixteenth century, the average length of human life was eighteen or twenty years, while in England at the present moment it is forty-four. This increased longevity is shown more strikingly in the fluctuations in the death rate of New York city for the last twenty years. In 1890 it was 25 out of each 1000, whereas now it is only 18."

" In his report to the Conservation Commission on "National Vitality", Irving Fisher, professor of economics at Yale University quotes the death rate of Massachusetts for 1865 and 1895 as illustrative of these two facts -- the decreased death rate for people under fifty and the increased death rate for people above that age:

Death Rate in Massachusetts per 1000 of Population in each Group.

	1865	1895	
5-9	9.6	6.2	Decrease
10-14	5.1	3.2	"
15-19	9.6	5.3	"
20-29	12.6	7.1	"
30-39	11.7	9.7	"
40-49	12	13	Increase
50-59	17	20	"
60-69	33	39	"
70z79	70	82	"
80- and upward	168	185	"

And I might quote farther but I believe it will be conceded by anyone who takes the trouble to investigate this subject thoroughly, that mortality rates have under gone very material changes in the last forty years and that any mortality table which can be constructed showing this change somewhat definitely will be of much value to students of social conditions and especially to those of life insurance problems, since that business depends so fundamentally upon mortality conditions. It seems to me that most writers emphasize too much the lack of medical advance in advanced ages as a cause for the increased death rate for those ages when the very fact that there is a decreased rate for younger ages would account for much. I make no claim that my final results should be accepted as representing these conditions exactly but they should show fairly accurately the trend in the mortality rates, while the rates themselves, in my judgment, will not be found to vary much from actual conditions.

Although statistics taken from the records of life insurance companies are far more accurate in regard to what data they do present, such data will fail to represent the mortality rates exactly as they exist at present, from the fact that the original material represents the history of lives beginning, in the majority of instances, several decades ago. By considering the quotient of deaths by the population as the prevalent rate of mortality for an age, we secure results in terms of present conditions. In the following I shall complete my table from age 0 to 105 for the benefit of those biologically inclined, in spite of the fact that the figures for the last ten years are of little worth for representing actual conditions and that the figures of the first ten years are very unreliable.

(A) CONSTRUCTION OF MORTALITY TABLE

(1) Source of Data

The data used in the construction of this table was taken from the United States census report for 1900. To get the most reliable figures, I chose those collected from the registration area, which for 1900 consisted of the District of Columbia and the nine states: Maine, Connecticut, Massachusetts, Michigan, New Hampshire, New Jersey, New York, Rhode Island, and Vermont. The registration area differs from the rest of the country in that the registration is done more carefully and accurately. In general a registration area is one where the deviations between official registration records and those due to census enumeration fall within ten per cent of the latter.

To lessen, as much as possible, the unavoidable influences of immigration, only the rural portion of the registration area was used. The registration in the rural portion has the reputation of being less reliable than that of the cities, but taking all factors into consideration, my choice of material seems best available. My figures for mortality were taken from Vital Statistics Part 2, but I am indebted to Mr Durand, the Director of the census, for the population data which he sent me, in its only available form, that of manuscript. The data is in quinquennial periods except the first five years where it is given for each age. The original data follows on the next page.

Agex	Lx	Dx
Under 1	143369	16836
1-2	133240	3598
2-3	138513	1683
3-4	141113	1139
4-5	140549	736
5-9	692469	2520
10-14	657610	1794
15-19	637893	2881
20-24	620865	3727
25-29	574559	3931
30-34	522055	3552
35-39	489081	3782
40-44	448606	3748
45-49	387655	3936
50-54	345949	4380
55-59	293916	5347
60-64	252629	6445
65-69	200287	7981
70-74	145809	8932
75-79	94701	9253
80-84	48712	7338
85-89	18087	3986
90-94	4582	1526
95----	1031	422

(2) Interpolation I assumed that my table should end at age 105 and therefore let $l_{105} = q_{105} = 0$

To interpolate within my quinquennial periods for each age both for l_x and d_x, I changed my data into columns of T_x where each t_x represents the total number at and above age x. For example, when applied to my l_x column, each t_x gave the number of persons living at age x <u>and above</u>. My method of interpolation was that described in detail by George King in the July number of J. I. A. for 1908 and referred to as Dr Sprague's Osculatory Method. The method requires that the two interpolation curves meeting at any point shall have the same gradient and radius of curvature, which was affected by giving them the same first and second differential coefficients. Geometrically, this required that our final curve should not only be smooth at the "joints" but that the curve throughout should present a smooth appearance.

Summing up the process, the equation was taken to be:

$$y_{2+x} = y_2 + ax + bx^2 + cx^3 + dx^4 + ex^5 + \text{------}$$

Inserting the values found for the constants deduced from the two given conditions, our equation becomes:

$$y_{2+x} = y_2 + x\Delta y_0 + \tfrac{1}{2}(3x + x^2)\Delta^2 y_0 + \tfrac{1}{6}(2x + 3x^2 + x^3)\Delta^3 y_0 - \tfrac{1}{24}(2x + x^2 - 2x^3 - x^4)\Delta^4 y_0 + \tfrac{1}{24}(7x^3 - 12x^4 + 5x^5)\Delta^5 y_0.$$

If we give to x the values $0/5, 1/5, 2/5 \text{----} 5/5$ and difference the resulting six equations five times, we form the leading differences by means of which to interpolate four values of y between the values of, say, y_2 and y_3, thus dividing the interval into five minor intervals. The resulting leading differences are as follows:

$$\left\{\begin{aligned}
\delta y &= \frac{\Delta y_0}{5} + 8\frac{\Delta^2 y_0}{5^2} + 11\frac{\Delta^3 y_0}{5^3} - 11\frac{\Delta^4 y_0}{5^4} + \frac{\Delta^5 y_0}{5^4} \\
\delta^2 y &= \frac{\Delta^2 y_0}{5^2} + 6\frac{\Delta^3 y_0}{5^3} + \frac{\Delta^4 y_0}{5^4} + 3\frac{\Delta^5 y_0}{5^4} \\
\delta^3 y &= \frac{\Delta^3 y_0}{5^3} + 4\frac{\Delta^4 y_0}{5^4} - 3\frac{\Delta^5 y_0}{5^4} \\
\delta^4 y &= \frac{\Delta^4 y_0}{5^4} - 2\frac{\Delta^5 y_0}{5^4} \\
\delta^5 y &= 5\frac{\Delta^5 y_0}{5^4}
\end{aligned}\right\}$$

First, I differenced my data five times, being careful of the signs of my differences; then I modified the differences by dividing those of each order by the appropriate power of 5 as shown above; then I formed the above functions themselves, carrying each process through for the entire set of T_x before going ahead to the next process. Each quinquennial interval will have its own set of subdivided differences derived from the modified differences of an age ten years younger. The interpolated values of y for each interval are formed by the continued addition of the differences in the usual way and there is a complete final check on the whole work because the next higher quinquennial value of y given in the original data must be reproduced at each stage. The work was performed to two more decimal places than are to be finally retained. For

the purpose of illustration, I give the work necessary for interpolating in the interval (35-40):

Differencing the proper interval five times as mentioned above:

25	3827659					
		-574559				
30	3253100		52504			
		-522055		-19530		
35	2731045		32974		27031	
		-489081		7501		-14056
40	2241964		40475		12975	
		-448606		20476		
45	1793358		60951			
		-387655				
50	1405703					

Modifying the leading differences above and forming our new leading leading differences represented on the preceding page by the Greek minor delta, I write them as above except that the sides are interchanged for sake of convenience as follows:

					2731045.00	35
				-100327.40		
			1138.50		2630717.60	36
		84.23		- 99188.90		
	88.23		1222.73		2531528.70	37
-112.45		172.46		- 97966.17		
	-24.22		1395.19		2433562.53	38
		148.24		- 96570.98		
			1543.43		2336991.55	39
				- 95027.55		
					2241964.00	40

After obtaining the complete tables of values for T_x for both l_x and d_x, I changed the results back to columns of the latter and formed the corresponding column of q_x.

By graphical interpolation, I found values for q_x for ages 5 to 9, 95 to 99, 101 to 104; q_{100} was found by using fourth differences which were formed from values for ages 80, 85, 90, 95 and 105 by means of the formula:

$$\Delta^4 u_{80} = \frac{1}{5}\left\{ u_{105} - u_{80} - 5\Delta u_{80} - 10\Delta^2 u_{80} - 10\Delta^3 u_{80} \right\}$$

By means of the values of q_x, I constructed the mortality table

itself, taking l_{10} = 100000, as is customary. The complete table from age 0 to 103, follows:

Age	Lx	Dx	Qx	Age	Lx	Dx	Qx
0	121593	14279	.1174313	53	73857	986	.0133458
1	107593	2898	.0270039	54	72871	1053	.0144469
2	104416	1269	.0121505	55	71818	1123	.0156436
3	103147	833	.0080715	56	70695	1202	.0170083
4	102314	536	.0052366	57	69493	1276	.0183661
5	101778	468	.0046	58	68217	1336	.0195864
6	101310	405	.0040	59	66881	1389	.0207649
7	100905	363	.0036	60	65492	1444	.0220520
8	100542	322	.0032	61	64048	1500	.0234251
9	100220	220	.0022	62	63548	1572	.0251328
10	100000	84	.0008408	63	60976	1672	.0274256
11	99916	227	.0022709	64	59304	1795	.0302705
12	99689	345	.0034573	65	57509	1914	.0332838
13	99344	369	.0037126	66	55595	2036	.0366137
14	98975	339	.0034274	67	53559	2149	.0401218
15	98636	360	.0036474	68	51410	2246	.0436838
16	98276	409	.0041615	69	49164	2334	.0474736
17	97867	452	.0046142	70	46830	2415	.0515753
18	97415	483	.0049620	71	44415	2489	.0560422
19	96932	506	.0052150	72	41926	2566	.0612140
20	96426	530	.0054956	73	39360	2649	.0672938
21	95896	555	.0057858	74	36711	2728	.0743162
22	95341	575	.0060356	75	33983	2785	.0819508
23	94766	594	.0062667	76	31198	2823	.0904921
24	94172	608	.0064542	77	28375	2821	.0994038
25	93564	621	.0066386	78	25554	2769	.1083712
26	92943	645	.0068309	79	22785	2682	.1176903
27	92298	641	.0069405	80	20103	2587	.1286742
28	91657	636	.0069433	81	17516	2485	.1418913
29	91021	625	.0068661	82	15031	2338	.1555672
30	90396	614	.0067894	83	12693	2339	.1842822
31	89782	600	.0066795	84	10354	1860	.1796806
32	89182	595	.0066728	85	8494	1639	.1929476
33	88587	604	.0068183	86	6856	1429	.2084757
34	87983	622	.0070683	87	5427	1232	.2269939
35	87361	641	.0073360	88	4195	1042	.2483121
36	86720	660	.0076117	89	3153	857	.2717653
37	86060	674	.0078292	90	2296	683	.2975888
38	85386	678	.0079423	91	1613	527	.3264427
39	84708	675	.0079661	92	1086	384	.3532220
40	84033	675	.0080311	93	702	264	.3766892
41	83358	676	.0081138	94	438	168	.3831325
42	82682	683	.0082574	95	270	107	.396
43	81999	700	.0085313	96	163	67	.4087511
44	81299	723	.0088928	97	96	42	.434
45	80576	749	.0093007	98	54	25	.469
46	79827	779	.0097629	99	29	15	.500
47	79048	809	.0102297	100	14	8	.5520352
48	78239	830	.0106125	101	6	4	.608
49	77409	850	.0109752	102	2	1	.686
50	76559	873	.0114050	103	1	1	.768
51	75686	896	.0118411	104	0		
52	74790	933	.0124718				

(B) Graduation by Makeham's Formula

In this section, it is our purpose to graduate or smooth our table from age 19 to 85. It is needless to go beyond these ages for graduation, for from our very method of interpolating into the quinquennial periods, all data within ten years of the years limiting our original data, (0 to 95) must be vitiated; moreover, data under ten years of age is unreliable

Therefore, I place the lower limit of the table at 19. I place it at 19 instead of 20 for adaptation to the quadrature formula mentioned below. To adapt Makeham's Formula, we find a function which will give the number living at any age x. This function is of the form:

$$l_x = K s^x g^{c^x}$$

whose four constants are to be determined by use of the data.

The method of determining these constants most, in the past is that described in the Text-book of the Institute of Actuaries. This method depends too much on empirical trials of different sets of the data and the success of such trials, in general, depends on the manner of representing all the data. The method lacks a great deal of being systematic and has little value from an analytic point of view.

The method to be used here is that of Moments, described by Karl Pearson in Philosophical Transactions of the Royal Society for 1895, Part 1 in an article entitled," Contributions to the Mathematical Theory of Evolution ", and applied by the same author in our own problem in Biometrika, Vol I. This latter method seems to be an improvement on the first mentioned one in that it gives thoroughly systematic plans of treating the problem. After making certain transformations and putting the function in logari-

thmic form, (designating the logarithms by capital letters):

$$L_x = K + S_1 x + G C^x \quad (\text{where } C = \varepsilon^{\frac{2n}{l}})$$

" We must now proceed to find the area and first three moments $A, A\mu_1, A\mu_2, A\mu_3$ of L_x about the middle of the range l. If we then equate these to the moments found from the table we shall have equations to determine K, S, G and n and therefore k, s, g, and c in a perfectly direct and systematic manner, using all the data provided. For example:

$$A = \int_{-\frac{1}{2}l}^{+\frac{1}{2}l} L\,dx = Kl + \frac{Gl}{2n}(\varepsilon^{n} - \varepsilon^{-n})$$

Likewise, evaluating

$$A\mu_1 = \int_{-\frac{1}{2}l}^{\frac{1}{2}l} Lx\,dx, \quad A\mu_2 = \int_{-\frac{1}{2}l}^{\frac{1}{2}l} Lx^2\,dx, \quad A\mu_3 = \int_{-\frac{1}{2}l}^{\frac{1}{2}l} Lx^3\,dx$$

and substituting in the following definitions:

$$\alpha_0 = \frac{A}{l} \qquad \alpha_1 = \frac{12A\mu_1}{l^2} \qquad \alpha_2 = \frac{12A\mu_2}{l^3} \qquad \alpha_3 = \frac{80A\mu_3}{l^4}$$

we have, finally:

$$\alpha_0 = K + G\frac{\sinh n}{n}$$

$$\alpha_1 = S + 6G\left\{\frac{\cosh n}{n} - \frac{\sinh n}{n^2}\right\}$$

$$\alpha_2 = K + 3G\left\{\frac{\sinh n}{n} - \frac{2\cosh n}{n^2} + \frac{2\sinh n}{n^3}\right\}$$

$$\alpha_3 = S + 10G\left\{\frac{\cosh n}{n} - \frac{3\sinh n}{n^2} + \frac{6\cosh n}{n^3} - \frac{6\sinh n}{n^4}\right\}$$

from which, is obtained β, defined as

$$\beta\{=(\alpha_3-\alpha_1)(\alpha_2-\alpha_0)\} = \frac{4\left(\frac{20\mu_3}{l^3} - \frac{3\mu_1}{l}\right)}{\frac{12\mu_2}{l^2} - 1}$$

which reduces to

$$\tanh n = \frac{2n^3 + 30n + 3\beta n^2}{\beta n^3 + 12n^2 + 3\beta n + 30}$$

and finally to

$$\varepsilon^{2n} - \frac{(\beta+2)n^3 + 3(\beta+4)n^2 + 3(\beta+10)n + 30}{(\beta-2)n^3 - 3(\beta-4)n^2 + 3(\beta-10)n + 30} = 0$$

This final equation was approximated by Newton's method which consists in abbreviating Taylor's series, where

$$f(x) = f(a+x) = f(a) + h f'(a) + \text{-----} = 0$$

where "a" represents a close trial to a root and h, the needed correction, is to be found by solving

$$h = -\frac{f(a)}{f'(a)}$$ which in this problem, becomes $$h = \frac{D\varepsilon^{2n} - N}{\frac{dN}{dn} - \frac{N}{D}\cdot\frac{dD}{dn} - 2\varepsilon^{2n} D}$$

where N stands for the numerator and equals $y_2 + y_1$ and D, the denominator equals $y_2 - y_1$ in

$$y_1 = 2n^3 + 3\beta n^2 + 30n$$

$$y_2 = \beta n^3 + 12n^2 + 3\beta n + 30$$

In the above solution, I shortened the work by dealing empirically with $f(a) \cdot D = D\varepsilon^{2n} - N$, until I found the largest "a" for which $D\varepsilon^{2n} - N$ is positive for the number of places with which I was then working; then I found h.

For close approximations of the area under the curve and its attendant moments, I used Weddle's Quadrature Formula for p elements (p here, being equal to eleven):

$$\int_{X_0}^{X_{6p}} z\,dx = \frac{3}{10}\left\{ \begin{aligned} & z_0 + z_2 + z_4 + \cdots z_{10} + \cdots z_{6p-2} + z_{6p} \\ & + 2(z_6 + z_{12} + \cdots z_{6p-6}) \\ & + 5(z_1 + z_5 + z_7 + \cdots z_{6p-1}) \\ & + 6(z_3 + z_9 + z_{15} \cdots + z_{6p-3}) \end{aligned} \right\}$$

The ordinates, of course, are $z = L = \log l_x$ for the area, xz for the first, $x^2 z$ for the second, and $x^3 z$ for the third moment, where attention must be paid to the sign of x. The values of constants auxiliary to the work were found to be as given on the next page.

$A =$ 315.05838411 $\log \varepsilon^{2n} =$ 1072.790800

$A\mu_1 =$ -261.45562645 $\log \varepsilon^{\frac{2n}{l}} =$ .04591689455

$A\mu_2 =$ 112021.86689402 $\varepsilon^{n} =$ 32.75348581

$A\mu_3 =$ -190550.78340751 $\varepsilon^{-n} =$ .03053110

$\alpha_0 =$ 4.77361188045454 $\frac{\sinh n}{n} =$ 4.689433452

$\alpha_1 =$ -.72026343374656 $\frac{\cosh n}{n} =$ 4.698184101

$\alpha_2 =$ 4.67576036789465 $\frac{\sinh n}{n^2} =$ 1.344058706

$\alpha_3 =$ -.80338733949188 $\frac{\cosh n}{n^2} =$ 1.346566768

$n =$ 3.489009393 $\frac{\sinh n}{n^3} =$ .3852264511

$n^2 =$ 12.173186544 $\frac{\cosh n}{n^3} =$ .3859452974

$n^3 =$ 42.472362195 $\frac{\sinh n}{n^4} =$ .1104114113

$n^4 =$ 148.186470641 $\beta =$ .8494902487526

The final constants were found to be:

G = -.02709948903

K = 4.90069313

S/l = -.002649892842

C = 1.111519011 $= \varepsilon^{\frac{2n}{l}}$

C^{-1} = .8996697223

The function thus takes the form:

$$Lx = 4.90069313 - .002649892842\,(x) - (.02709948903)(1.111519011)^{x}$$

which gives us the logarithm of the number living at age x.

The table on the next page gives (A) a column of calculated ordinates, (B) a column of original observed ordinates, (C) one of the deviations from calculated minus observed values, (D) deviations obtained by Pearson in his problem of graduation in the article in Biometrika, quoted above and here used for comparison.

Pearson's mean deviation is .00116

My own is ,00046944

Pearsonæs mean deviation by the Average method is .00126

	(A)	(B)	(C)	(D)
19	4.98731221	4.98646717	.00084504	
20	4.98457005	4.98419415	.00037590	
21	4.98181761	4.98180049	.00001712	
22	4.97905572	4.97927970	-.00022598	
23	4.97627711	4.97665255	-.00037544	
24	4.97348639	4.97392179	-.00043540	
25	4.97067995	4.97110878	-.00042883	.00184
26	4.96785605	4.96821669	-.00036064	.00083
27	4.96501275	4.96519229	-.00017954	.00056
28	4.96214789	4.96216564	-.00001775	.00005
29	4.95925905	4.95914160	.00011745	.00008
30	4.95634355	4.95614921	.00019434	-.00013
31	4.95339845	4.95318928	.00020917	-.00001
32	4.95042042	4.95027721	.00014321	-.00034
33	4.94740579	4.94736999	.00003580	-.00032
34	4.94435050	4.94439877	-.00004827	-.00037
35	4.94124999	4.94131760	-.00006761	-.00034
36	4.93809923	4.93811927	-.00002004	-.00060
37	4.93489261	4.93480134	.00009127	-.00068
38	4.93162391	4.93138667	.00023724	-.00056
39	4.92828620	4.92792443	.00036177	-.00023
40	4.92487178	4.92444987	.00042191	.00012
41	4.92137211	4.92094729	.00042482	.00000
42	4.91777767	4.91741097	.00036670	-.00001
43	4.91407790	4.91380856	.00026934	-.00008
44	4.91026103	4.91008520	.00017583	-.00040
45	4.90631404	4.90620570	.00010834	-.00038
46	4.90222239	4.90214981	.00007258	-.00034
47	4.89796995	4.89789089	.00007906	-.00048
48	4.89353881	4.89336778	.00017103	-.00016
49	4.88890902	4.88879146	.00011756	-.00018
50	4.88405844	4.88399625	.00006219	.00001
51	4.87896243	4.87901555	-.00005312	.00039
52	4.87359364	4.87384353	-.00024989	.00077
53	4.86792164	4.86839166	-.00047002	.00056
54	4.86191262	4.86255473	-.00064211	-.00005
55	4.85552899	4.85623331	-.00070432	-.00040
56	4.84872897	4.84938870	-.00065973	.00046
57	4.84146614	4.84194106	-.00047492	.00075
58	4.83368888	4.83389262	-.00020374	.00071
59	4.82533982	4.82530276	.00003706	.00047
60	4.81635520	4.81618825	.00016695	-.00050
61	4.80666413	4.80650557	.00015856	.00021
62	4.79618783	4.79621343	-.00002560	.00055
63	4.78483875	4.78515893	-.00032018	.00073
64	4.77251954	4.77308399	-.00056445	.00095
65	4.75912201	4.75973582	-.00061381	.00112
66	4.74452592	4.74503573	-.00050981	.00138
67	4.72859761	4.72883246	-.00023485	.00148
68	4.71118849	4.71104760	.00014089	.00087
69	4.69213345	4.69164721	.00048624	.00123
70	4.67124891	4.67052416	.00072475	.00212
71	4.64833087	4.64752967	.00080120	-.00161
72	4.62315280	4.62248343	.00066937	-.00504
73	4.59546186	4.59505509	.00040677	-.00306
74	4.56497866	4.56479621	.00018245	-.00460
75	4.53139152	4.53126172	.00012980	.00121

76	4.49435429	4.49412675	.00022754	.00202
77	4.45348221	4.45293587	.00054634	.00266
78	4.40834763	4.40745889	.00088874	-.00007
79	4.35847521	4.35764903	.00082618	-.00080
80	4.30333657	4.30326087	.00007570	-.00029
81	4.24256219	4.24343494	-.00087275	-.00332
82	4.17484604	4.17698787	-.00214183	-.00306
83	4.10011580	4.10356428	-.00344848	-.00343
84	4.01734724	4.01510816	.00223908	.00694
85	3.92564391	3.92911226	-.00346835	-.00578

Changing my columns of logarithms over to columns of l and d themselves and adding the resulting columns of the original data, we have: (the calculated columns come first)

Age	L	D	L-L'		Age	L'	D
19	97121	612	189		19	96932	506
20	96509	609	73		20	96426	530
21	95900	609	4		21	95896	555
22	95291	607		50	22	95341	575
23	94684	606		82	23	94766	594
24	94078	606		94	24	94172	608
25	93472	606		92	25	93564	621
26	92866	606		77	26	92943	645
27	92260	607		38	27	92298	641
28	91653	607		4	28	91657	636
29	91046	609	25		29	91021	625
30	90437	612	41		30	90396	614
31	89825	614	43		31	89782	600
32	89211	617	29		32	89182	595
33	88594	621	7		33	88587	604
34	87973	625		10	34	87983	622
35	87348	632		13	35	87361	641
36	86716	638		4	36	86720	660
37	86078	645	18		37	86060	674
38	85433	654	47		38	85386	678
39	84779	664	71		39	84708	675
40	84115	675	82		40	84033	675
41	83440	688	82		41	83358	676
42	82752	702	70		42	82682	683
43	82050	718	51		43	81999	700
44	81332	736	33		44	81299	723
45	80596	756	20		45	80576	749
46	79840	778	13		46	79827	779
47	79062	802	14		47	79048	809
48	78260	830	21		48	78239	830
49	77430	860	21		49	77409	933
50	76570	893	11		50	76559	873
51	75677	930		9	51	75686	896
52	74747	970		43	52	74790	933
53	73777	1014		80	53	73857	986
54	72763	1061		108	54	72871	1053
55	71702	1114		116	55	71818	1123
56	70588	1171		107	56	70695	1202
57	69417	1232		76	57	69493	1276
58	68185	1298		32	58	68217	1336
59	66887	1360	6		59	66881	1389
60	65517	1446	25		60	65492	1444
61	64071	1527	23		61	64048	1500
62	62544	1613		4	62	62548	1572
63	60931	1704		45	63	60976	1672
64	59227	1799		77	64	59304	1795
65	57428	1898		81	65	57509	1914
66	55530	2000		65	66	55595	2036
67	53530	2103		29	67	53559	2149

Age	L	D	L-L'	Age	L'	D
68	51427	2208	17	68	51410	2246
69	49219	2311	55	69	49164	2334
70	46908	2411	78	70	46830	2415
71	44497	2506	82	71	44415	2489
72	41991	2585	65	72	41926	2566
73	39406	2680	46	73	39360	2649
74	36726	2733	15	74	36711	2728
75	33993	2779	10	75	33983	2785
76	31214	2803	16	76	31198	2823
77	28411	2805	36	77	28375	2821
78	25606	2778	52	78	25554	2769
79	22828	2721	43	79	22785	2682
80	20107	2626	4	80	20103	2587
81	17481	2524	35	81	17516	2485
82	14957	2365	74	82	15031	2338
83	12592	2184	101	83	12693	2339
84	10408	1982	54	84	10354	1860
85	8426	____	68	85	8494	1639

It might be well to notice that my mean (L-L') = 54-
and that Glover in his graduation of the
American Experience Table =132-